



June 29, 2005

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Tool - Models - Theory

non-equilibrium thermodynamics (NLTE)
radiation hydrodynamics

theoretical framework

Boltzmann - equation

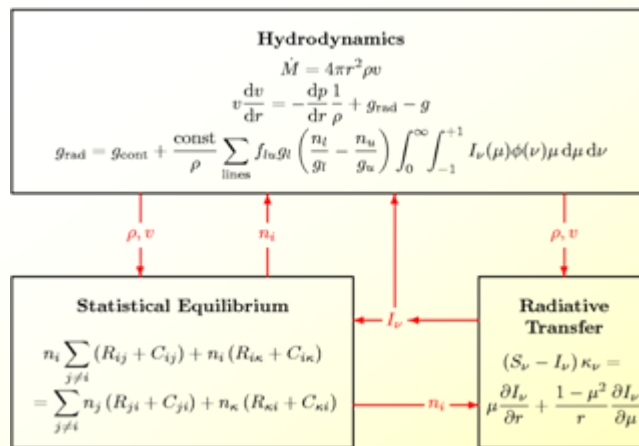
$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{1}{m} \mathbf{F} \cdot \nabla_{\mathbf{v}} f = \left[\frac{\partial f}{\partial t} \right]_{ic}$$

time development of the distribution functions

Simplifying assumptions for calculating consistent models of

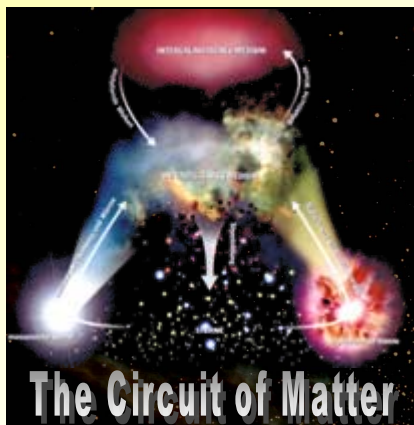
Expanding Atmospheres of Hot Star:
homogeneity, stationarity, spherical symmetry

Nonetheless, the solution is a non-trivial problem due to back-reactions which influence the results of the solution primarily



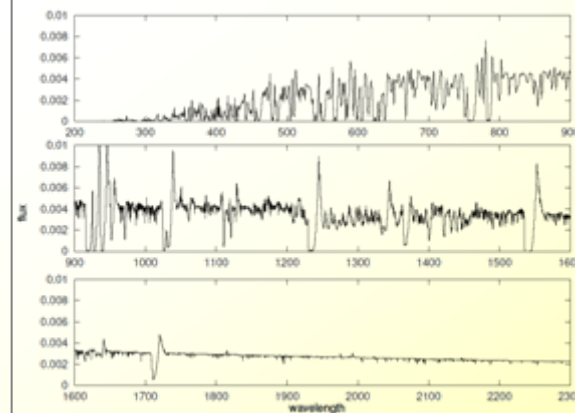
Motivation Observations

development of consistent atmospheric models



Application Spectral Diagnostics

development of detailed synthetic spectra



Tool - Models - Theory

non-equilibrium thermodynamics (NLTE)
radiation hydrodynamics

Boltzmann - equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{1}{m} \mathbf{F} \cdot \nabla_{\mathbf{v}} f = \left[\frac{\partial f}{\partial t} \right]_{ic}$$

time development of the distribution functions

Motivation
Observations

Application
Spectral Diagnostics

Hot Stars

$T_{\text{eff}} = 15000 - 100000 \text{ K}$

show wind features

Expanding
Atmospheres

characterized by

high radiation energy
density

sub-group
Low Mass Stars

0.4 - 1.44 M_{\odot}

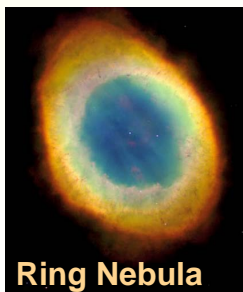
CSPN (SN Ia)

old, 10^{10} yr

luminous, $10^4 L_{\odot}$

rare, evolution

radiation, UV/EUV



Ring Nebula

sub-group
Massive Stars

20 - 150 M_{\odot}

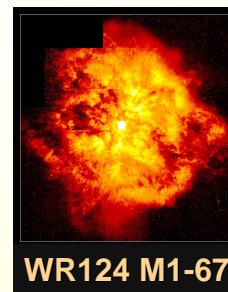
O/B (WR SN II)

young, 10^6 yr

luminous, $10^6 L_{\odot}$

rare, IMF

radiation, UV/EUV



WR124 M1-67

$$10^{-4} M_{\odot}/\text{yr} \geq \dot{M} v_{\infty}$$

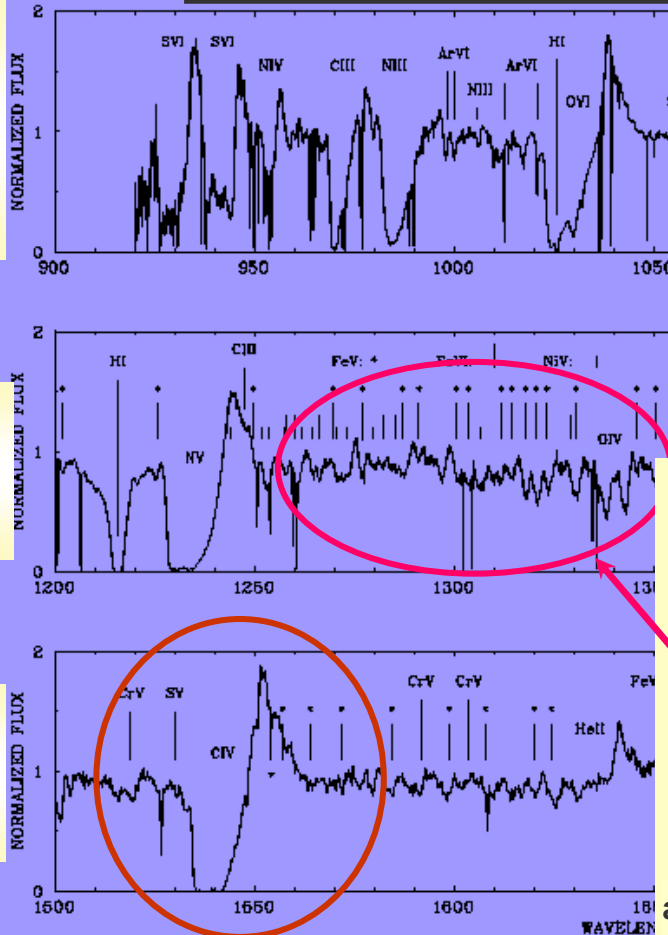
$$\dot{M} v_{\infty} \leq 4000 \text{ km/s}$$

In this talk focus on the status of UV Spectral Diagnostics !

Emergent UV Spectra of O-type Stars are Old Fashioned !

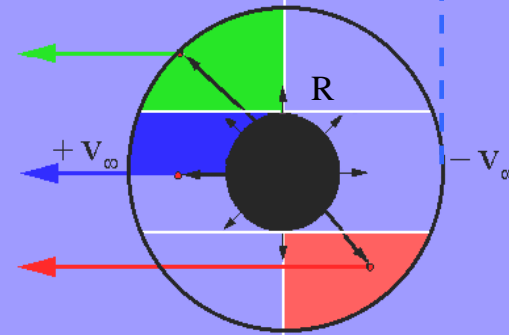
Walborn, Nichols-Bohlin, Panek, 1985, NASA Ref. Publ. 1155; Morton and Underhill, 1977, Ap. J. Suppl. 33, 83

Typical UV Spectrum of a Massive Star - ζ Pup (O4If)



$$\dot{M} \geq 10^{-8} M_{\odot}/\text{yr}$$

$$\frac{r}{R} \approx 100$$



Expanding atmospheres have a pronounced effect on the emergent spectra of hot stars!

momentum transfer produces the outflow !
→ consistent models

UV spectral diagnostics should be based on realistic models !

Structure of the lines depends on:

$\log g, T_{\text{eff}}$ and R



M, L, R, Z

all fundamental parameters can be deduced

Signatures of P-Cygni lines are characterized by the outflow

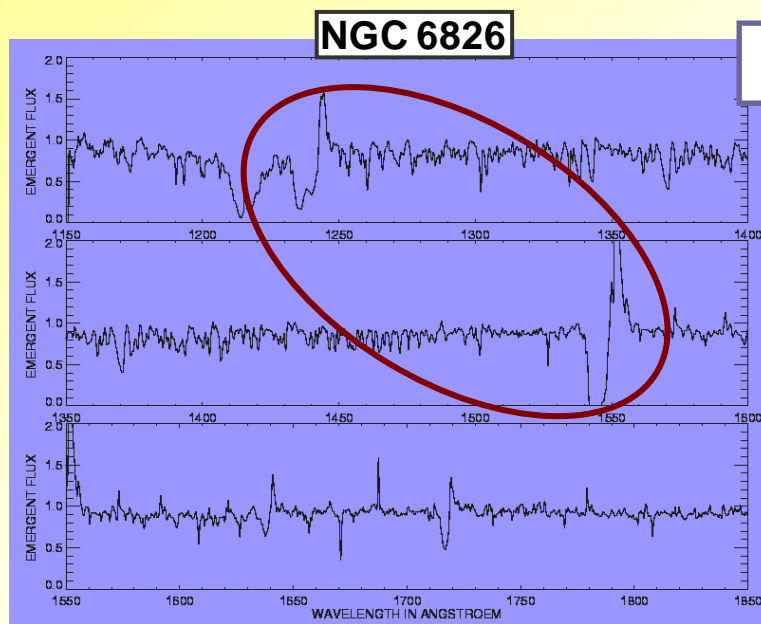
SVI, CIII, NIII, OVI, PV, NV, OIV, OV, SiIV, CIV, HeII, NIV

Hundreds of strongly wind contaminated lines are formed throughout the expanding atmosphere

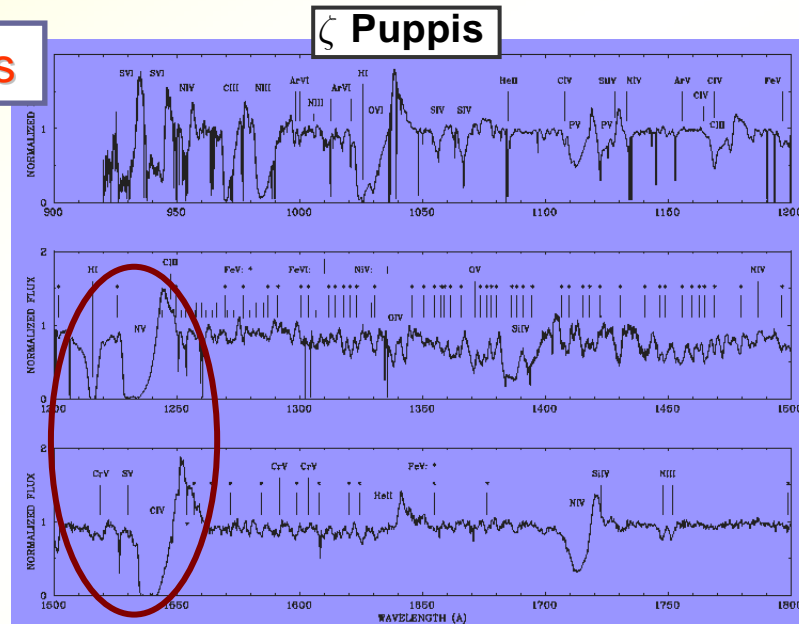
FeV, NiV, FeIV, FeVI, CrV, ArV, ArVI

Synthetic UV Spectra of Hot Stars - Still Relevant ?

Realistic Synthetic UV Spectra of O-type Stars a Diagnostic Tool with great Astrophysical Potential ?



Hot Stars



spectra of O-type CSPN's

similar to

spectra of massive O-type stars

Spectra contain information about stellar and wind parameters and abundances

superb sensitivity and spectral resolution provided for > 25 years
but most work done concentrated on qualitative results and arguments

Powerful Tool for modeling realistic synthetic spectra for
diagnostic issues is still in **development** !

Focus will be on the **ionizing radiation** of hot stars !

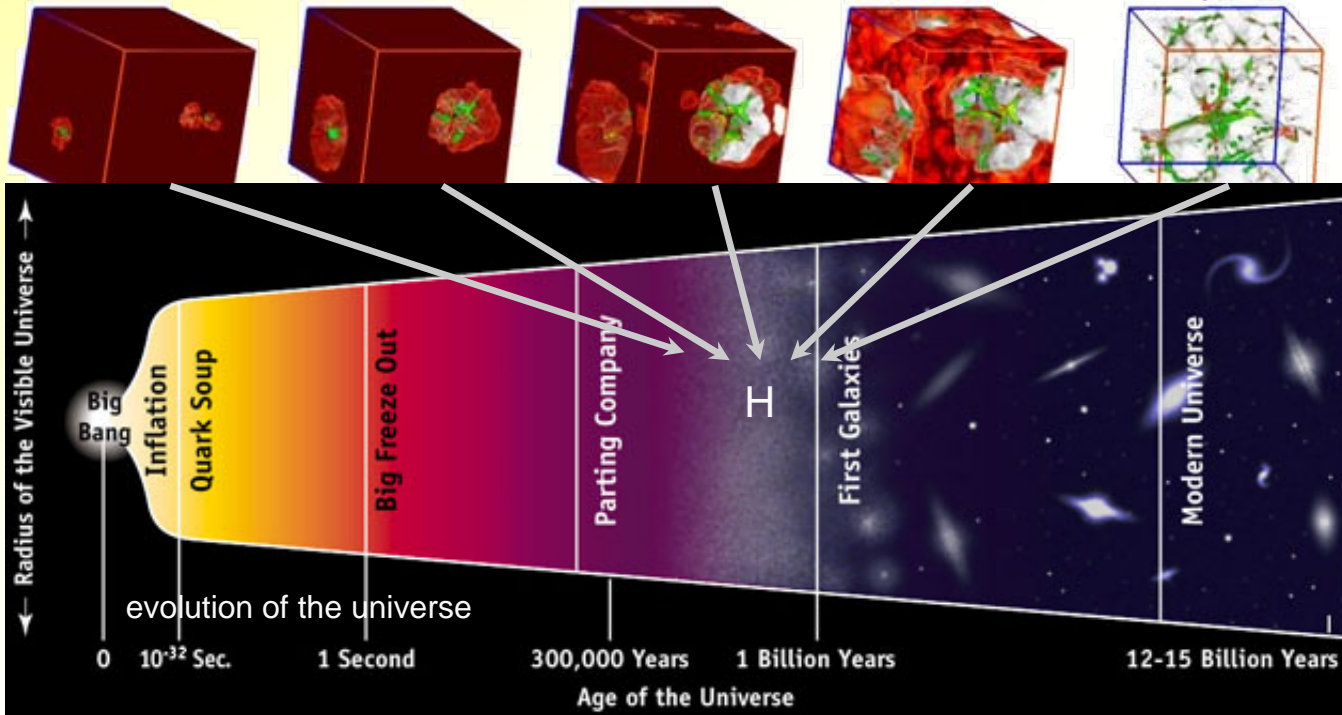
Motivation: Reionization of the universe

End of Cosmic Dark Ages

absence of Gunn-Peterson trough in the spectra of high-redshift quasars implies that **the universe was reionized at a redshift of $z \sim 6$**

Fan et al., 2000, AJ, 120, 1167

Gnedin, 2000, ApJ, 535, 530



What caused the reionization of the universe ?

Are population III stars ($Z < 10^{-3}$) the source for the reionization ?

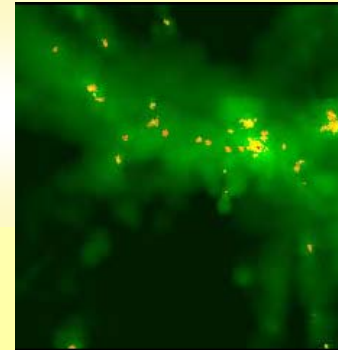
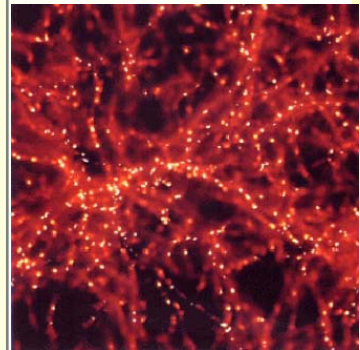
Carr et al., 1984, ApJ 277, 445

this is just the case if the **IMF of the First Stars** is different from what we know

Bromm et al., 1999, ApJ, 527, L5

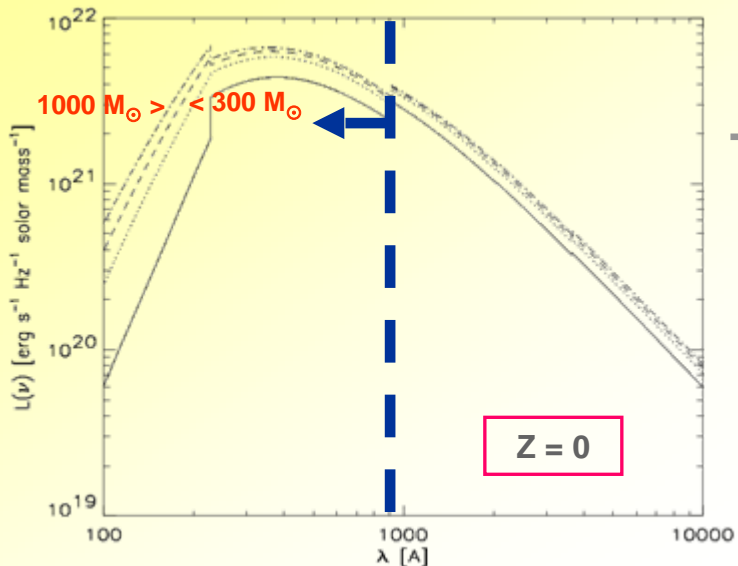
Numerical simulation of the first stars in the universe

Davé, Katz, Weinberg, 2002, ApJ, 579, 23



Reionization : Ionization Efficiency of the First generations of Stars

In the **Early Universe Population III stars** favoured the formation of very **Massive Stars**
 $M > 100 M_{\odot}$



Normalized **spectral energy distribution** in the continuum
 100 – 1000 M_{\odot} $Z=0$
 Per unit mass the spectra attain
 an universal form for $M > 300 M_{\odot}$
 Bromm, Kudritzki et al., 2001, ApJ, 540, 68

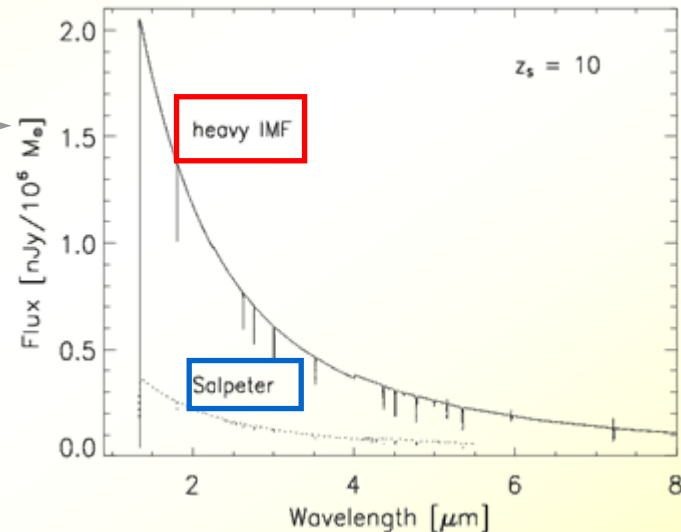
The enormous amount of **UV and EUV radiation** of these stars can change the status of the universe to **become reionized again**

Realistic Spectral Energy Distributions

of **O-type Stars** are also required

to analyze excited **HII regions and Planetary Nebulae**

IMF becomes top-heavy



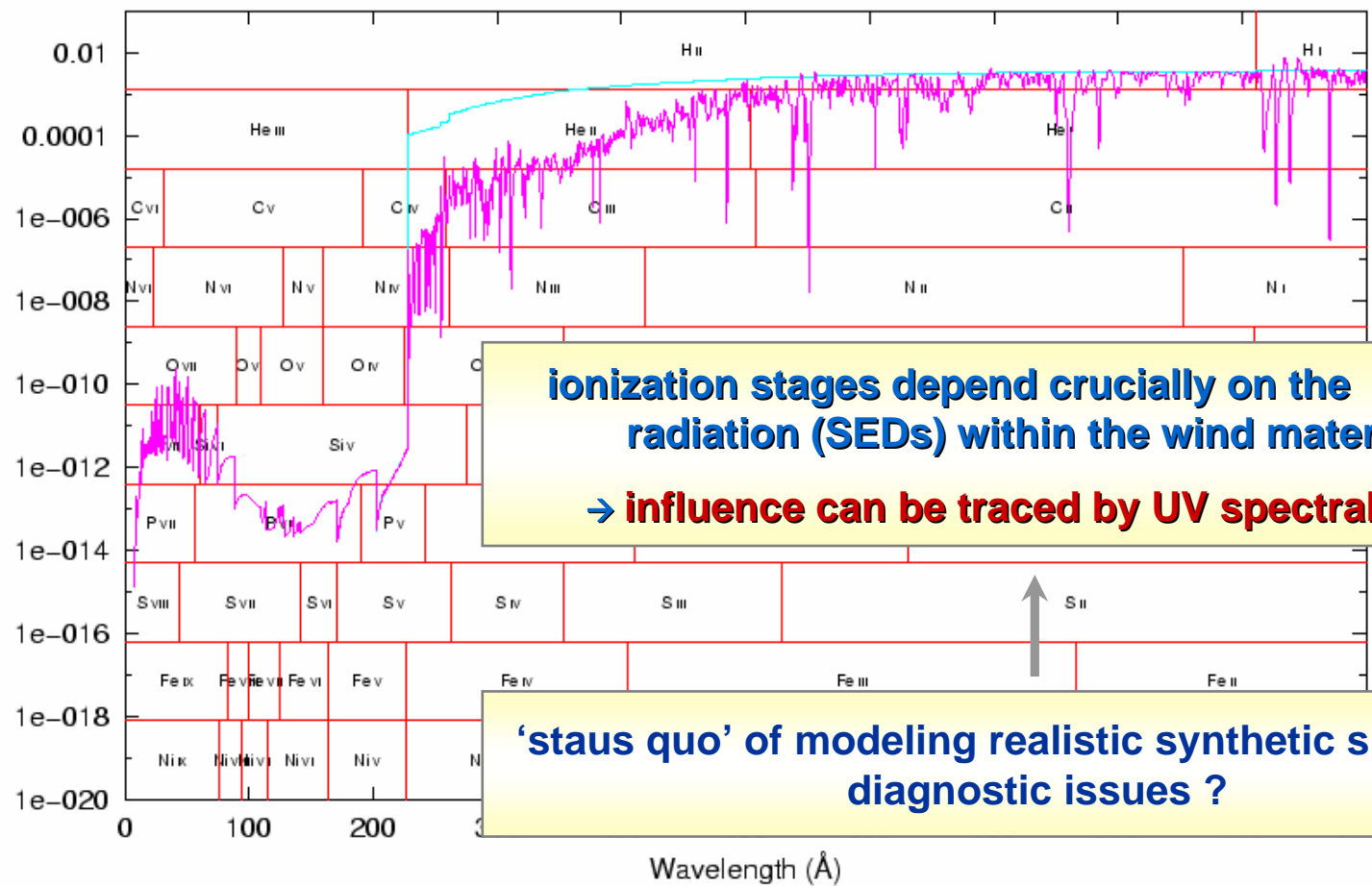
Predicted flux from a Population III star cluster at $z=10$
 A flat universe with $\Omega_{\Lambda} = 0.7$ is assumed.
 The cutoff is due to complete Gunn-Peterson absorption (1965, ApJ, 142, 1633).
 The flux is larger by an order of magnitude for the case of the **heavy IMF**
 Bromm, Kudritzki et al., 2001, ApJ, 540, 687

the flux of **very massive stars** can contribute the decisive part to the unexplained **deficit of ionizing photons**

Ionizing flux of ζ Puppis (O4If)

small vertical lines indicate the **ionization edges** for all important ions !

UV Spectra of O Stars:
test involves hundreds of spectral signatures of different ionization stages



ionization stages depend crucially on the ionizing radiation (SEDs) within the wind material !
→ **influence can be traced by UV spectral lines !**

'staus quo' of modeling realistic synthetic spectra for diagnostic issues ?

Concept for consistent atmospheric models of hot stars

theoretical basis

Lucy L. B., Solomon P., 1970, ApJ 159, 879
 Castor J.I., Abbott D.C., Klein R.: 1975, ApJ 195, 157

key aspects of theoretical activity

basic theoretical ideas

Milne E.A., 1926, MNRAS 86, 459
 Sobolev V., 1957, Sov. A&A J. 1, 678

Hydrodynamics

$$\dot{M} = 4\pi r^2 \rho v$$

$$v \frac{dv}{dr} = -\frac{dp}{dr} \frac{1}{\rho} + g_{\text{rad}} - g$$

$$g_{\text{rad}} = g_{\text{cont}} + \frac{\text{const}}{\rho} \sum_{\text{lines}} f_{lu} g_l \left(\frac{n_l}{g_l} - \frac{n_u}{g_u} \right) \int_0^\infty \int_{-1}^{+1} I_\nu(\mu) \phi(\nu) \mu d\mu d\nu$$

aspects of radiative transfer

Rybicki, G.B., 1971, JQSRT 11, 589
 Hummer, D.G., Rybicki, G.B., 1985, ApJ 293, 258

Rate Equations

$$n_i \sum_{j \neq i} (R_{ij} + C_{ij}) + n_i (R_{ik} + C_{ik})$$

$$+ n_i R_{ik}^K$$

Radiative Transfer

$$(S_\nu - I_\nu) \kappa_\nu =$$

$$\mu \frac{\partial I_\nu}{\partial r} + \frac{1 - \mu^2}{r} \frac{\partial I_\nu}{\partial \mu}$$

X-rays and wind shocks

Cassinelli, J., Olson, G., 1979, ApJ 229, 304
 Lucy L. B., White, R., 1980, ApJ 241, 300
 Owocki S., Castor, J., Rybicki, G. 1988, ApJ 335, 914

Shock Physics

$$S_\nu^S = f \frac{\rho^2}{4\pi \kappa_\nu} \Lambda_\nu(v)$$

K-Shell Ionization

$$\kappa_\nu^K = \sum_{Z, x, i} n_i \sigma_{ik}^K$$

Energy Equation

$$v \frac{de}{dr} + pv \frac{d}{dr} \left(\frac{1}{\rho} \right) =$$

$$\frac{1}{\rho} \int_0^\infty 4\pi \kappa_\nu (J_\nu - S_\nu) d\nu$$

basis of our theoretical framework

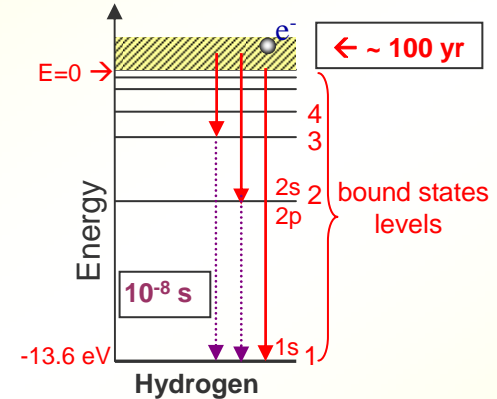
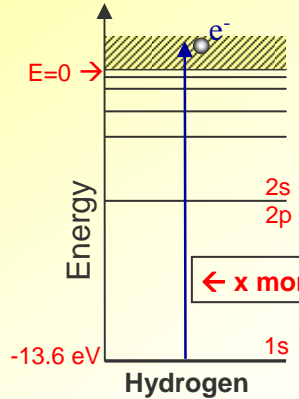
Pauldrach, A.W.A., Puls, J., Kudritzki, R.P., 1986, A&A, 164, 86; Pauldrach, A.W.A., 1987, A&A, 183, 295
 Puls, J., Pauldrach, A.W.A., 1990, PASPC 7, 203; Pauldrach, A.W.A. et al., 1994, A&A, 283, 525
 Feldmeier, A. et al., 1997, A&A 320, 899; Pauldrach, A.W.A. et al., 1998, ASPCS 131, 258
 Pauldrach, A.W.A., Hoffmann, T.L., Lennon, M., 2001, A&A, 375, 161

Theory: Motivation

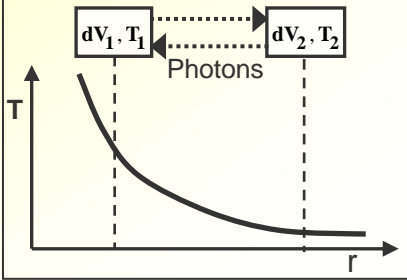
in GNs radiative processes are not balanced in detail !

→ thermodynamic equilibrium can not be established !

main processes:



Disadvantage of **NLTE**:
non local problem!



just ground states are occupied
→ no thermodynamic equilibrium (TE)
instead

Non-Equilibrium Thermodynamics (NLTE)

described by a well defined mixture of
macroscopic and microscopic physics

Transport Theory
Thermodynamics
Quantum Mechanics

Advantage of TE:

In **TE** state of the gas completely described by
2 macroscopic variables

→ **local** problem!

$$n_i^* = n_i(\rho, T)$$

n_i^* = occupation numbers of bound levels
(number of particles per cm^3)

n_i follows from **Boltzmann statistics**

$$n_1 4\pi \int_{\nu_{1k}}^{\infty} \frac{\alpha_{1k}(\nu)}{h\nu} J_\nu d\nu = n_P n_e \alpha_R$$

ionization equation

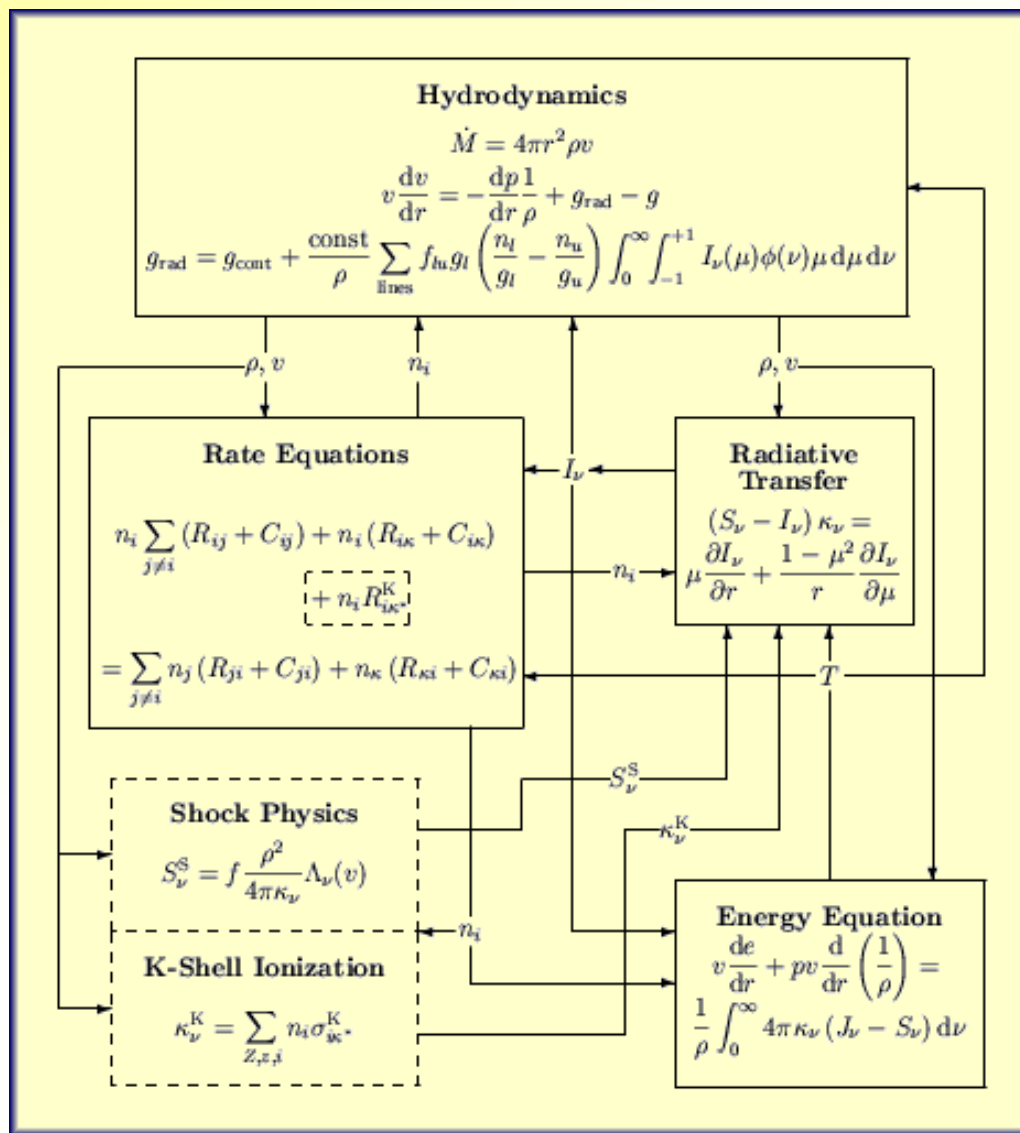
$$\left[\frac{1}{r^2} \frac{\partial}{\partial r} \right] r^2 J_\nu = -n_1 \alpha_{1k} J_\nu (+\eta_\nu)$$

the equation of transfer in spherical symmetry

Integro-Differential Equation

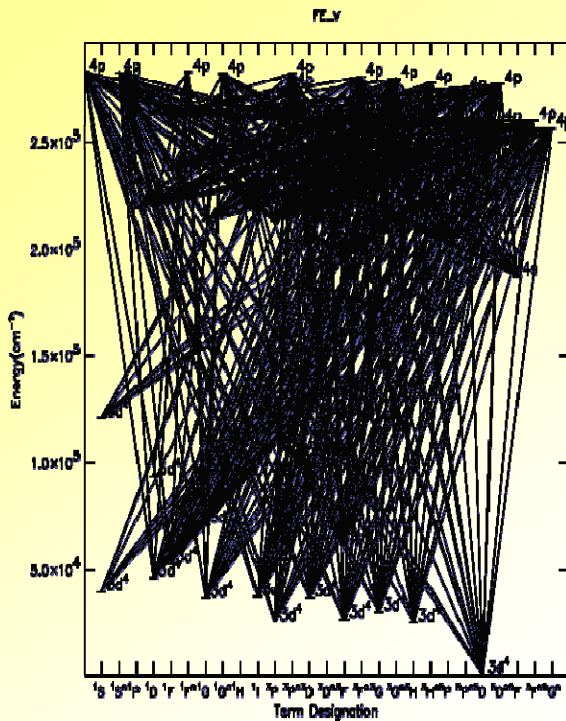
Constraints for the parameters and the abundances are obtained

T_{eff}	Z
L	M
\dot{M}	v_{∞}



observed spectra

Points which are of special importance for the complete procedure



well-elaborated
atomic models
are required
in order to avoid
GIGO

Rate Equations

$$n_i \sum_{i \neq j} (R_{ij} + C_{ij}) + n_i (R_{ik} + C_{ik}) = \sum_{i \neq j} n_j (R_{ji} + C_{ji}) + n_k (R_{ki} - C_{ki})$$

	I	II	III	IV	V	VI	VII	VIII
1	H_I							
2	He_I	He_II						
6	C_I	C_II	C_III	C_IV	C_V			
7	N_I	N_II	N_III	N_IV	N_V	N_VI		
8	O_I	O_II	O_III	O_IV	O_V	O_VI		
9	F_I	F_II	F_III	F_IV	F_V	F_VI		
10	Ne_I	Ne_II	Ne_III	Ne_IV	Ne_V	Ne_VI		
11	Na_I	Na_II	Na_III	Na_IV	Na_V	Na_VI		
12	Mg_I	Mg_II	Mg_III	Mg_IV	Mg_V	Mg_VI		
13	Al_I	Al_II	Al_III	Al_IV	Al_V	Al_VI		
14	Si_I	Si_II	Si_III	Si_IV	Si_V	Si_VI		
15	P_I	P_II	P_III	P_IV	P_V	P_VI		
16	S_I	S_II	S_III	S_IV	S_V	S_VI	S_VII	
17	Cl_I	Cl_II	Cl_III	Cl_IV	Cl_V	Cl_VI		
18	Ar_I	Ar_II	Ar_III	Ar_IV	Ar_V	Ar_VI	Ar_VII	Ar_VIII
19	K_I	K_II	K_III	K_IV	K_V	K_VI		
20	Ca_I	Ca_II	Ca_III	Ca_IV	Ca_V	Ca_VI		
22	Ti_I	Ti_II	Ti_III	Ti_IV	Ti_V			
23	V_I	V_II	V_III	V_IV	V_V			
24	Cr_I	Cr_II	Cr_III	Cr_IV	Cr_V	Cr_VI		
25	Mn_I	Mn_II	Mn_III	Mn_IV	Mn_V	Mn_VI		
26	Fe_I	Fe_II	Fe_III	Fe_IV	Fe_V	Fe_VI	Fe_VII	Fe_VIII
27	Co_I	Co_II	Co_III	Co_IV	Co_V	Co_VI	Co_VII	
28	Ni_I	Ni_II	Ni_III	Ni_IV	Ni_V	Ni_VI	Ni_VII	Ni_VIII
29	Cu_I	Cu_II	Cu_III	Cu_IV	Cu_V	Cu_VI		
30	Zn_I	Zn_II	Zn_III					

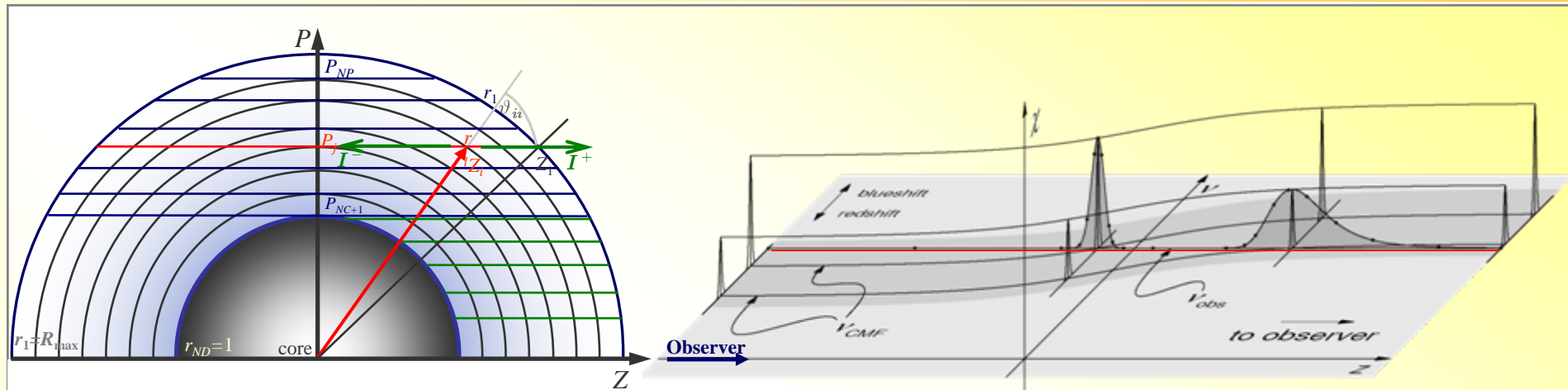
atomic data status:
excellent good poor bad

NLTE Models require atomic data for
 lines, collisions, ionization, recombination
Essential for occupation numbers, line blocking, line force
Accurate atomic models have been included

- 26 elements
- 149 ionization stages
- 5,000 levels (+ 100,000)
- 20,000 diel. rec. transitions
- 4 10⁶ b-b line transitions
- Auger-ionization

recently improved models are based on **Superstructure**
 Eisner et al., 1974, CPC 8,270

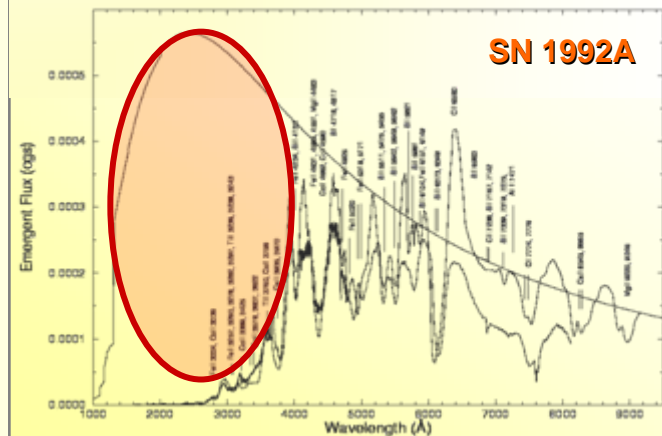
General principle of using adaptive grids also for problems involving radiative transfer



Radiative Transfer

$$(S_\nu - I_\nu)\chi_\nu = \mu \frac{\partial I_\nu}{\partial r} + \frac{1 - \mu^2}{r} \frac{\partial I_\nu}{\partial \mu}$$

a very close spatial resolution is required in order to resolve the spectral lines shifted into the line of sight!

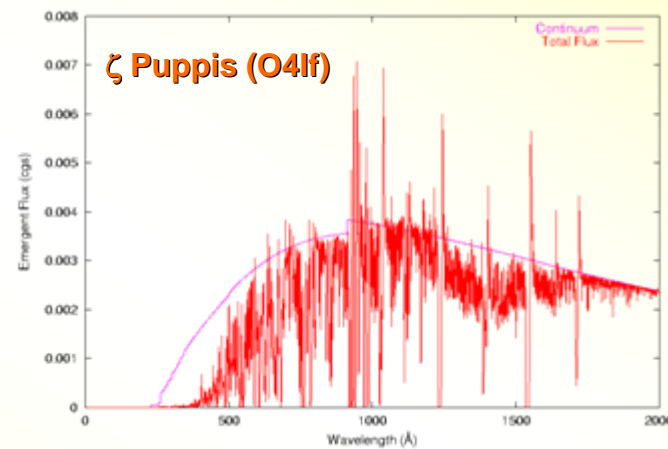


EUV line blocking and blanketing:

drastic effects on ionization excitation and emergent flux

reason:

the velocity field shifts at different radii up to 1000 spectral lines into the line of sight at the observer's frequency



Solution of this system presents the required Tool for calculating synthetic spectra

O supergiant exemplary model

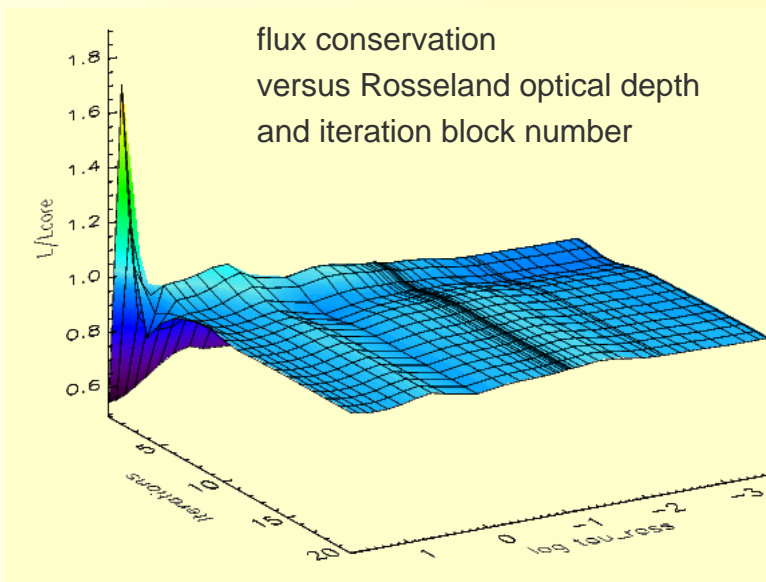
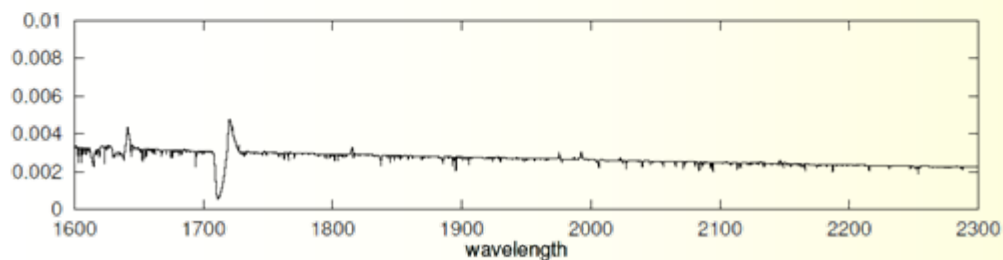
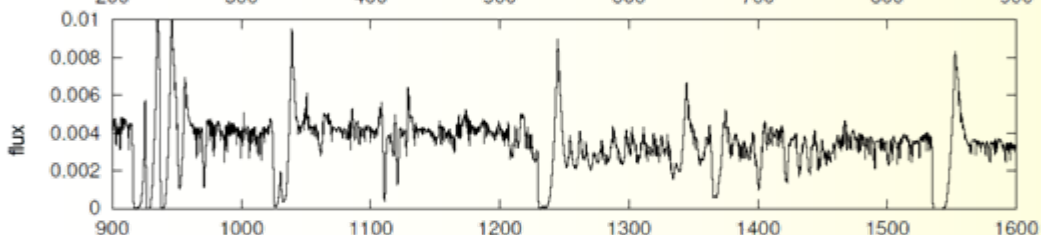
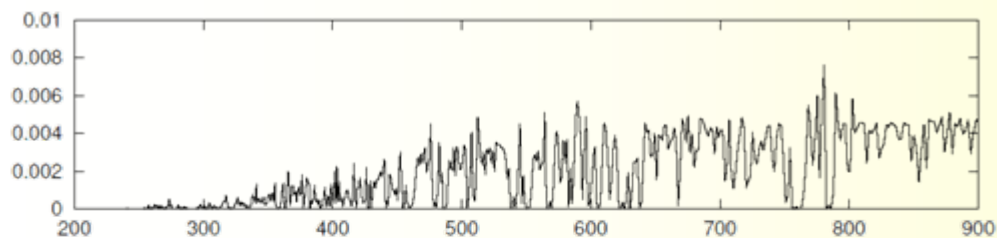
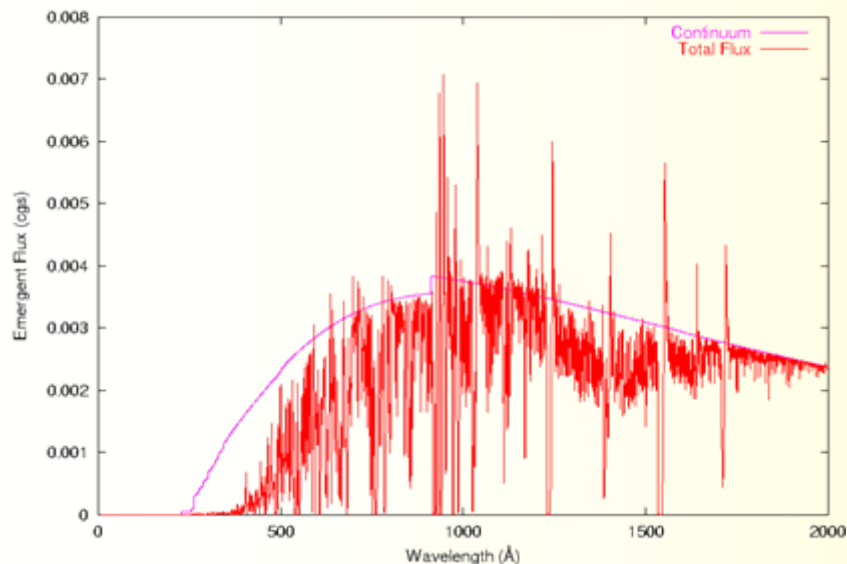
$$T_{\text{eff}} = 45000 \text{ K}$$

$$\log g = 3.6$$

$$R = 18 R_{\odot}$$

models lead to realistic results ?

Strategy for quantitative spectral UV analysis



flux conservation is on the 1% level

Detailed analysis of the hot O-supergiant ζ Puppis – final step

observed UV high resolution spectra can be regarded as being reproduced in total

ζ Puppis

Copernicus —
IUE —
model —

Values Determined:

stellar parameters

$$T_{\text{eff}} = 40 \text{ kK}$$

$$\log g = 3.40$$

$$R/R_{\odot} = 28.$$

$$V_{\text{rot}} / (\text{km/s}) = 220.$$

wind parameters

$$\frac{\dot{M}}{10^{-6} M_{\odot} / \text{yr}} = 13.7$$

$$V_{\infty} / (\text{km/s}) = 2120.$$

abundances

$$C/C_{\odot} = 1.50$$

$$N/N_{\odot} = 5.00$$

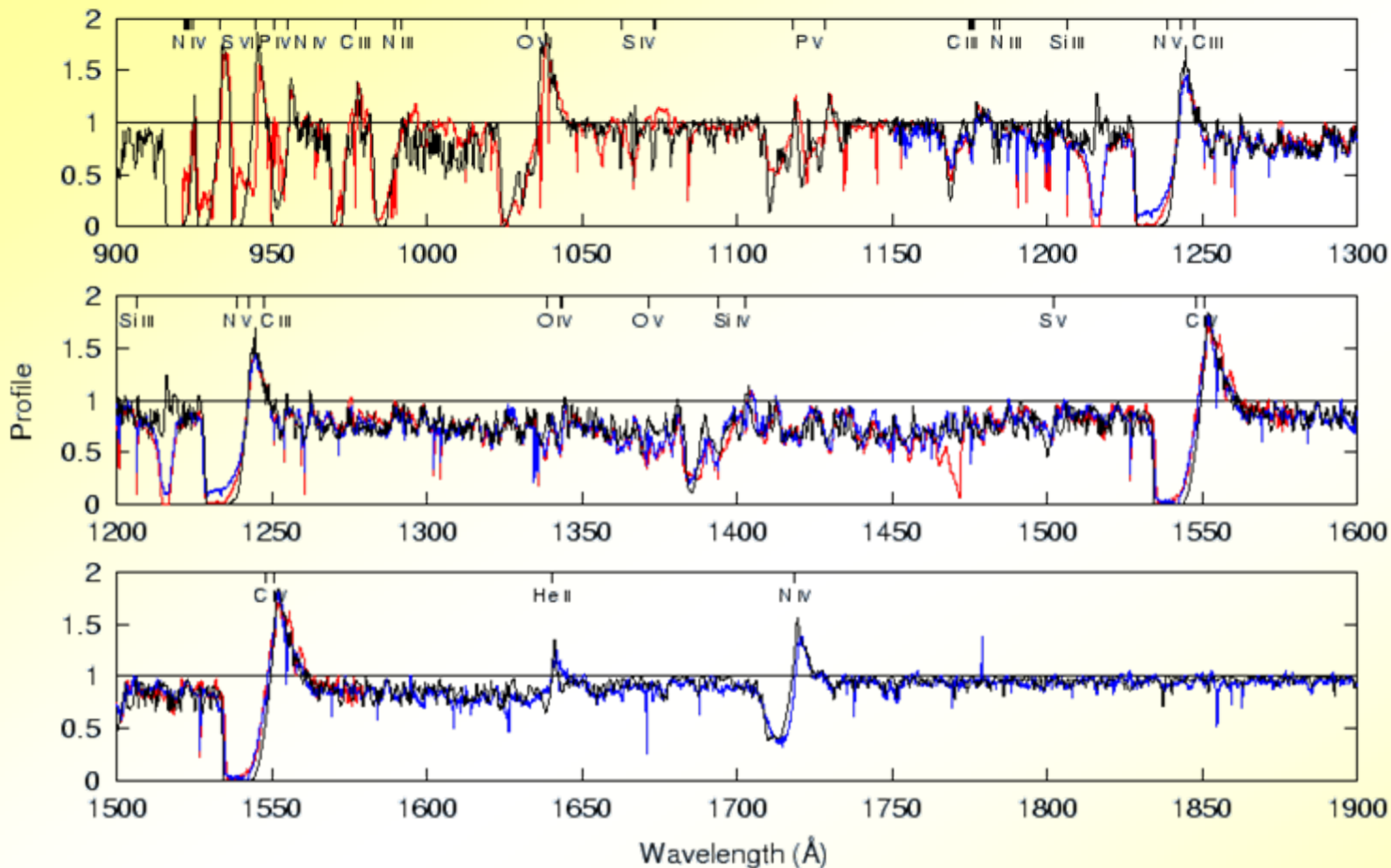
$$O/O_{\odot} = 0.10$$

$$\text{Si}/\text{Si}_{\odot} = 7.00$$

$$\text{S}/\text{S}_{\odot} = 0.50$$

$$\text{Fe}/\text{Fe}_{\odot} = 2.00$$

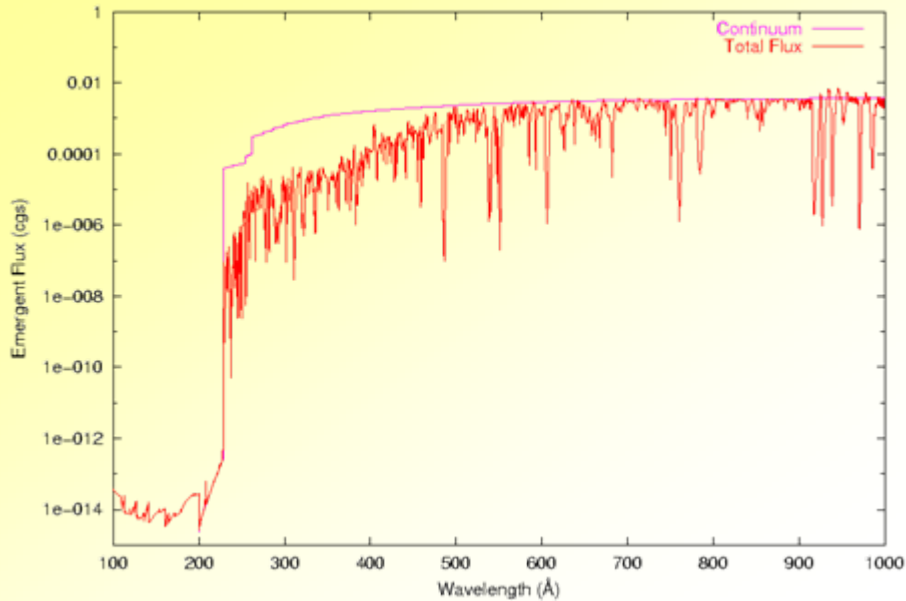
$$\text{Ni}/\text{Ni}_{\odot} = 2.00$$



consistent treatment of expanding atmospheres along with spectrum synthesis techniques allow the determination of **stellar parameters, wind parameters, and abundances**

present method of quantitative spectral UV analyses of hot stars leads to realistic models !

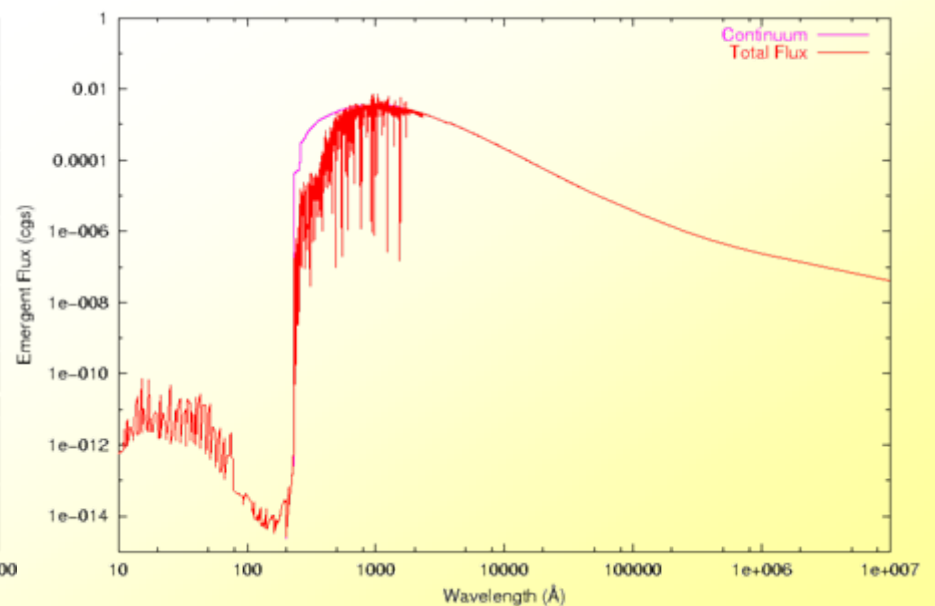
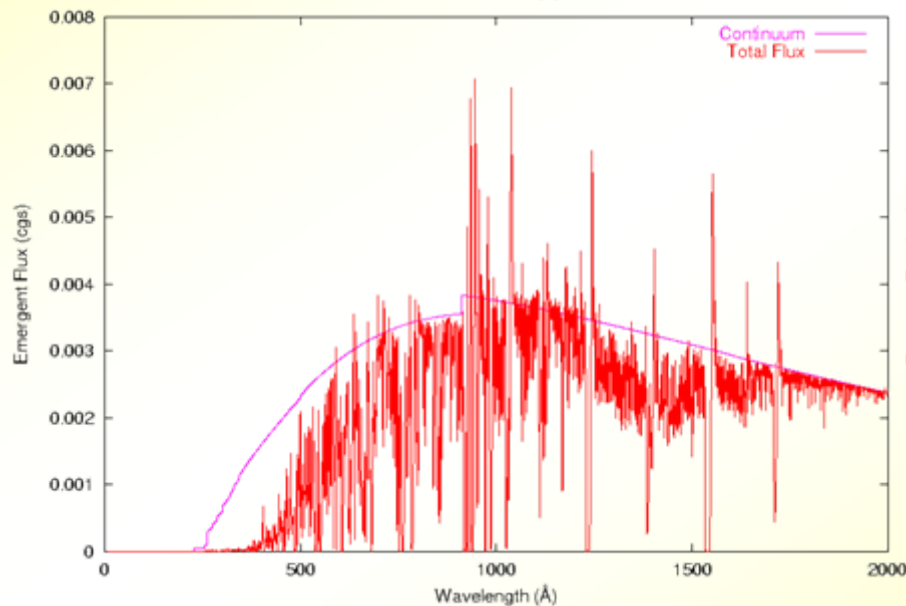
Realistic Spectral Energy Distribution of ζ Puppis



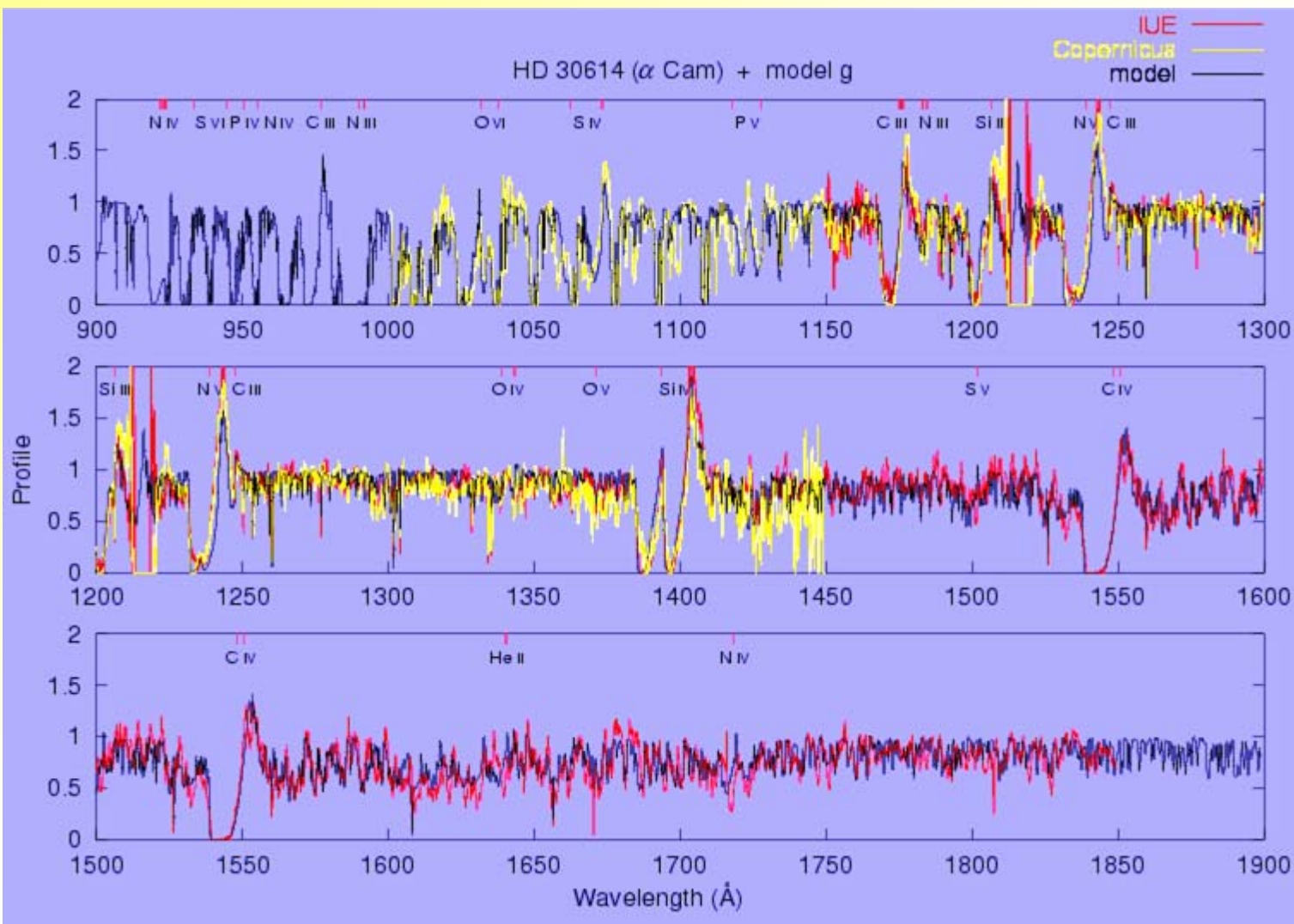
quantitative spectral UV analyses:
ultimate test for the accuracy of
theoretical ionizing fluxes



Realistic Spectral Energy Distributions



The method also works for cool O-supergiants - α Cam



$T_{\text{eff}} = 29 \text{ kK}$
 $\log g = 3.00$
 $R/R_{\odot} = 30$

$\frac{\dot{M}}{10^{-6}M_{\odot}/\text{yr}} = 5.0$
 $V_{\infty}/(\text{km/s}) = 1500.$

$C/C_{\odot} = 0.05$

$N/N_{\odot} = 1.00$

$O/O_{\odot} = 0.30$

$P/P_{\odot} = 0.05$

$S/S_{\odot} = 1.00$

$Fe/Fe_{\odot} = 1.50$

$Ni/Ni_{\odot} = 1.50$

Spectrum synthesis technique allows the determination of all required parameters

T_{eff} (within 1000 K), $\log g$ (within 0.05), R (within 1.0),
 and the abundances (within a range of 20%)

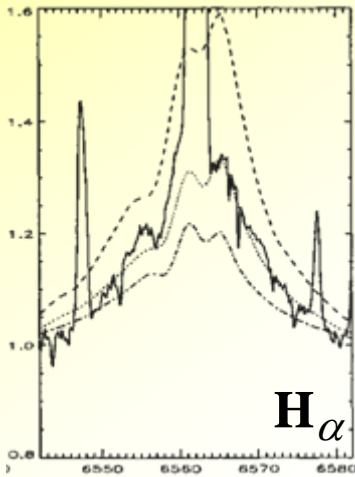
Wind momentum luminosity relation - significance of the dynamical parameters

winds driven by radiation → **mechanical momentum** of wind flow
mostly a function of **photon momentum**

$$\dot{M} v_{\infty} R^{1/2} \propto L^{3/2}$$

$$\dot{M}^{obs} (R/R_{\odot})^{0.5}$$

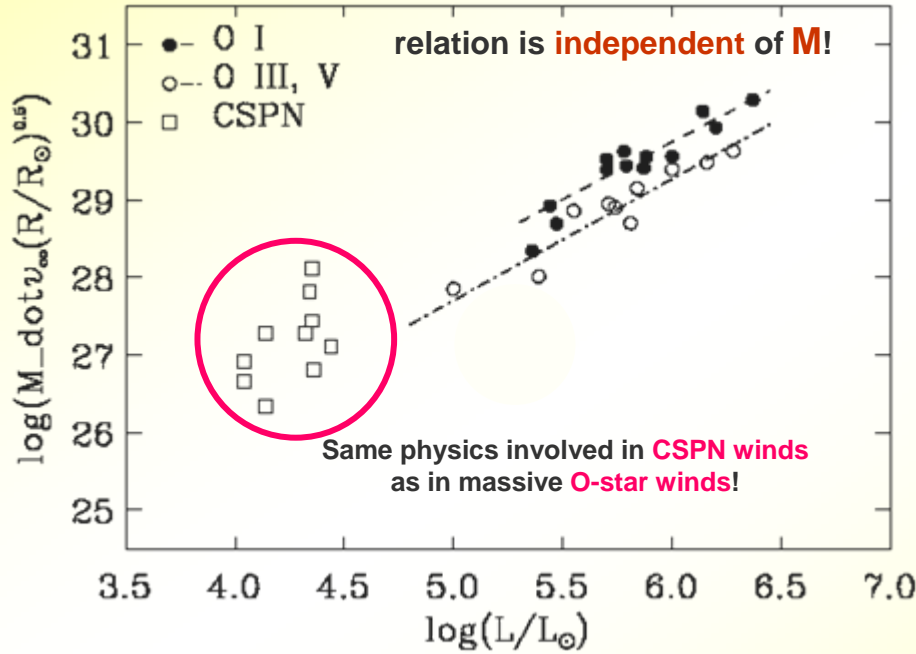
NGC 6826



$$\dot{M} = 3.5 * 10^{-7} M_{\odot}/yr$$

$$\dot{M} = 2.5 * 10^{-7} M_{\odot}/yr$$

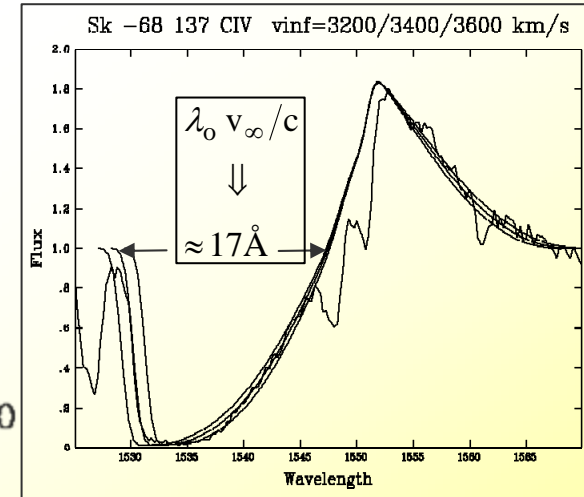
$$\dot{M} = 1.9 * 10^{-7} M_{\odot}/yr$$



Puls et al., 1996, A&A 305; Kudritzki et al., 1997, IAU Symp. 180

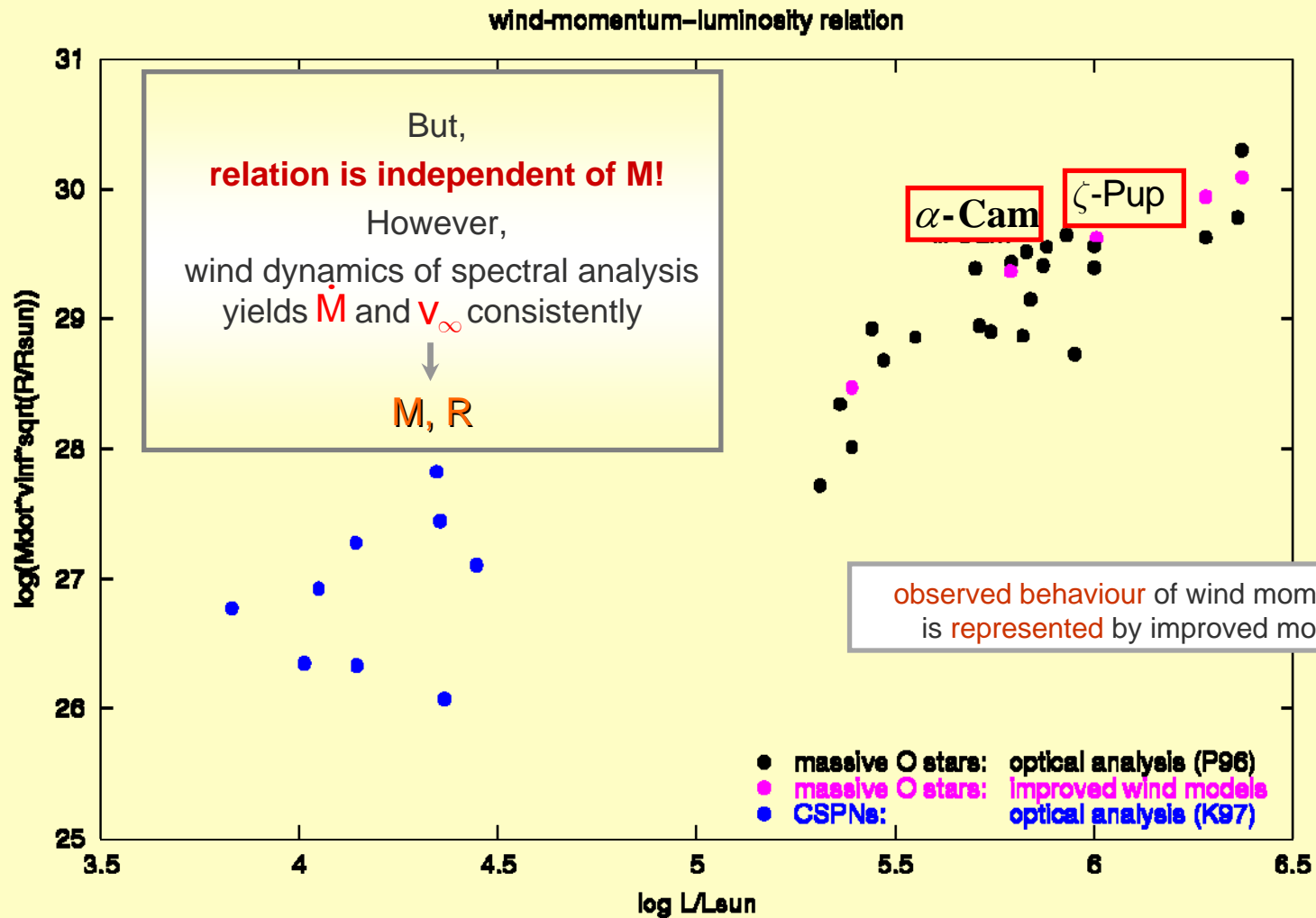
$$v_{\infty}^{obs}$$

CIV ($\lambda\lambda 1548, 1551 \text{\AA}$)



v_{∞} can be measured directly

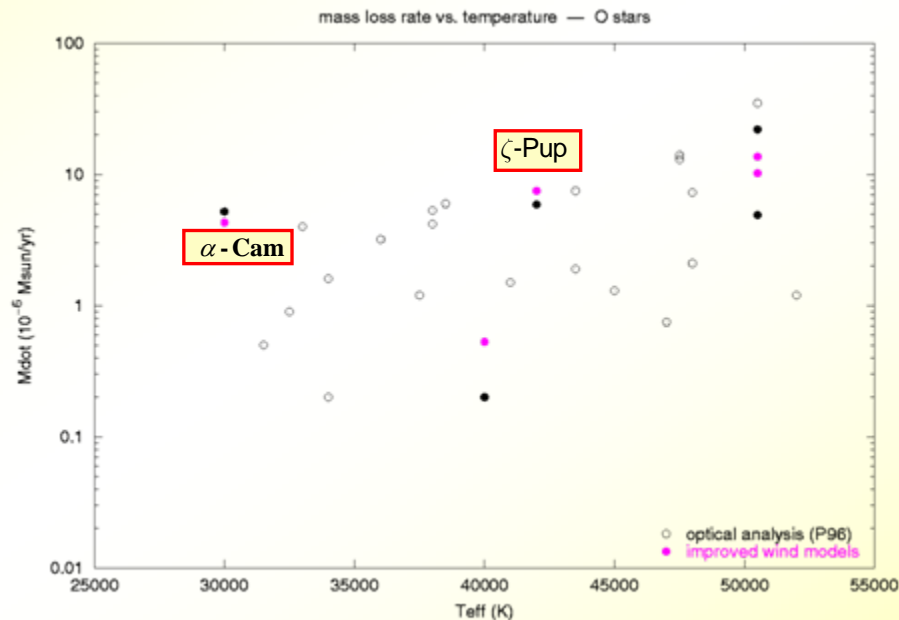
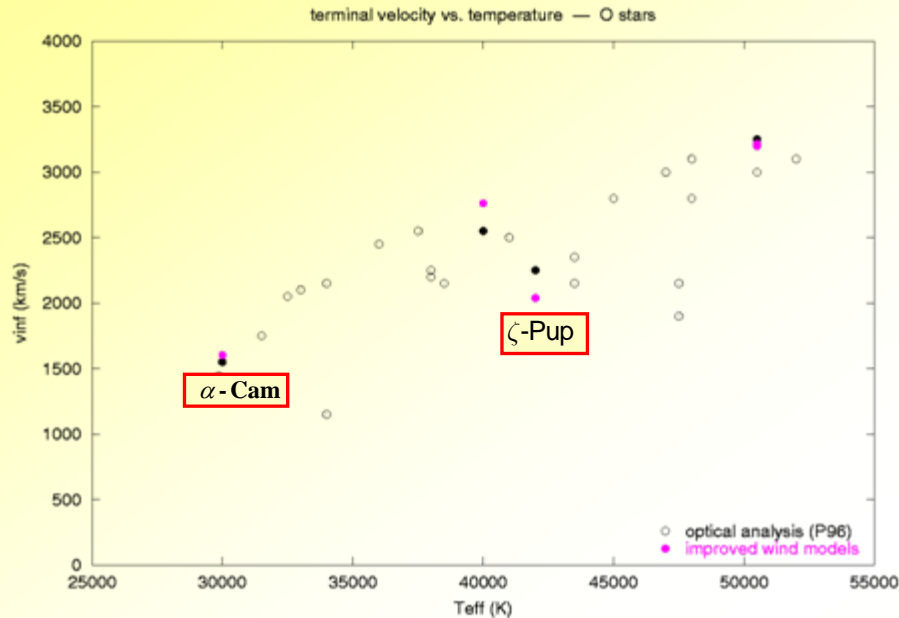
Wind momentum luminosity relation of massive O-stars



$$\dot{M} v_\infty R^{1/2} \propto L^{3/2}$$

observed values from Puls et al., 1996, A&A 305; Kudritzki et al., 1997, IAU Symp. 180

Wind properties of massive O-stars



Pauldrach, Hoffmann, Mendez, 2004, A&A, 419, 1111

$$v_{\infty} \propto v_{\text{esc}} = \left(\frac{2GM}{R} (1 - \Gamma) \right)^{1/2}$$

predicted terminal velocities
of improved models
agree within 10% with observed values

predicted mass-loss-rates
of improved models
agree within a factor smaller than 2 with
observed optical values

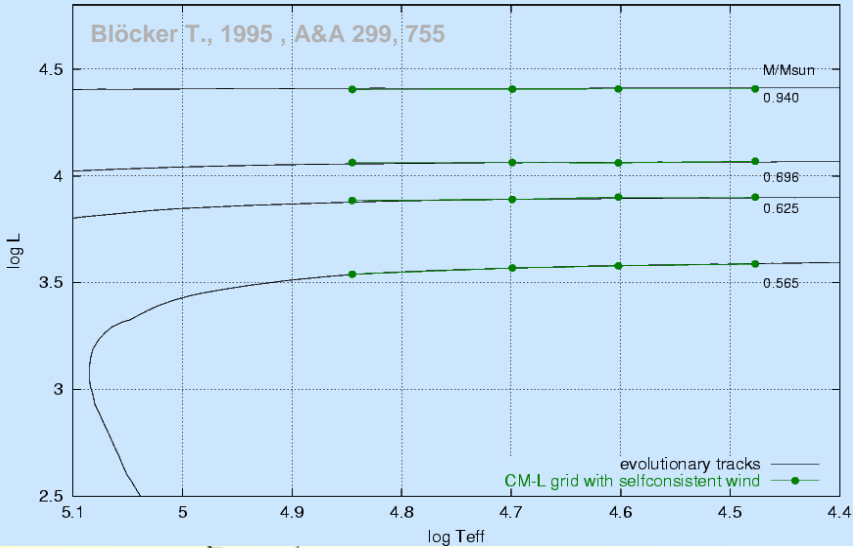
$$\dot{M} \propto L$$



M, R

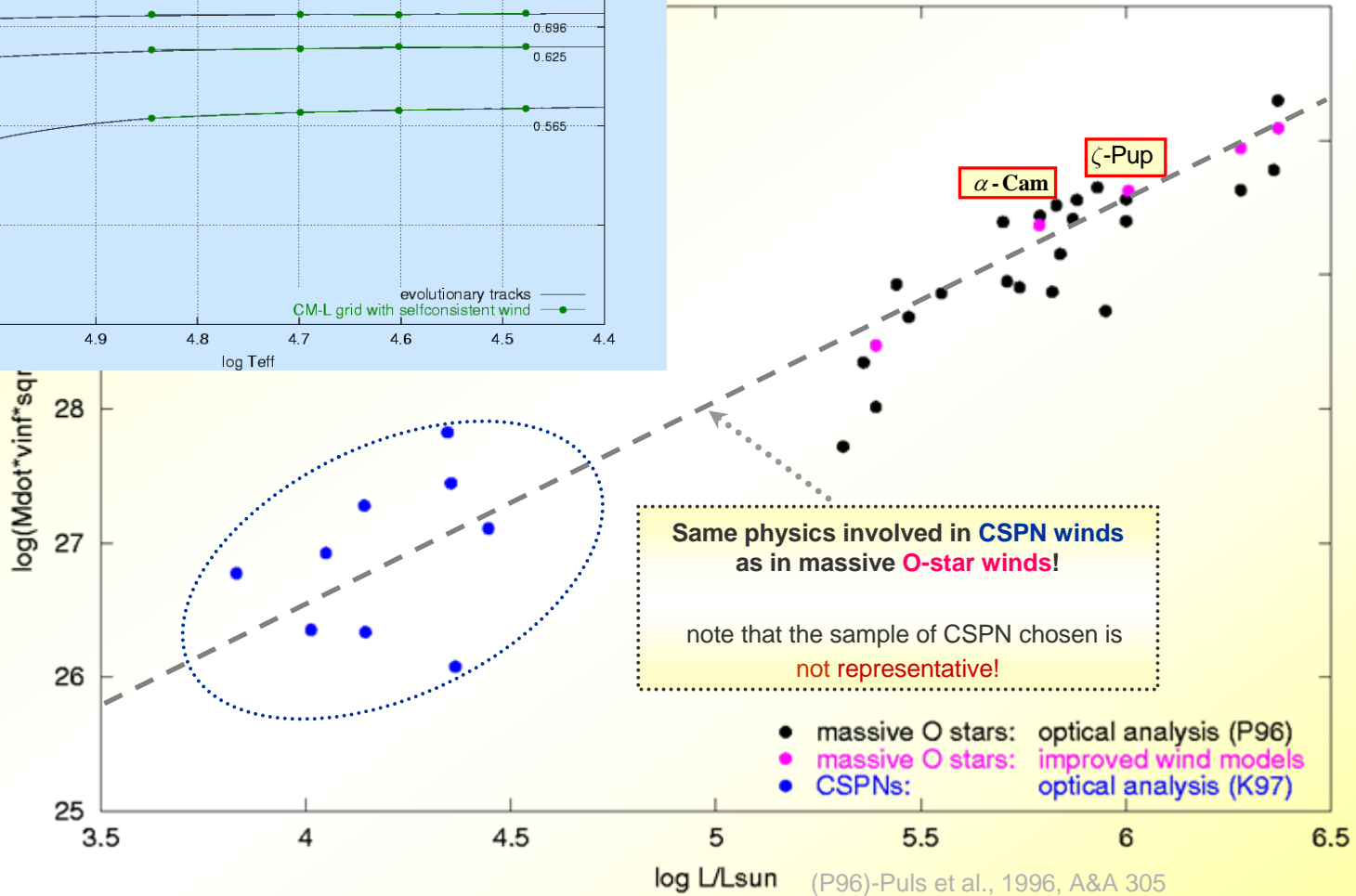
Wind properties of CSPN's

Luminosity vs. temperature CSPNs



model grid calculated along evolutionary tracks based on the **core-mass-luminosity relation**

lum-luminosity relation

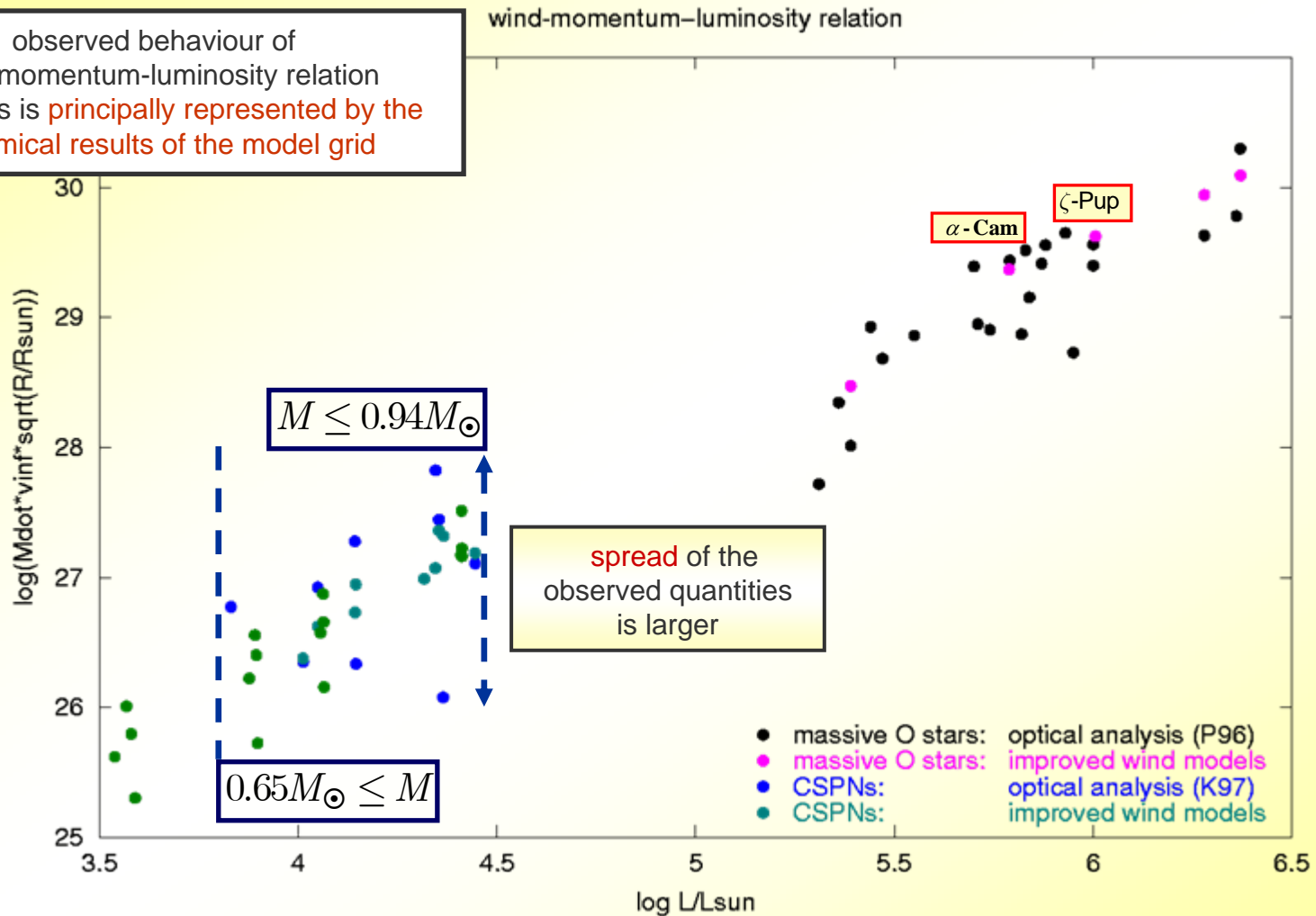


$$\dot{M} v_{\infty} R^{1/2} \propto L^{3/2}$$

Wind properties of CSPN's

model grid calculated along evolutionary tracks based on the **core-mass-luminosity relation**

observed behaviour of wind-momentum-luminosity relation of CSPNs is **principally represented by the dynamical results of the model grid**

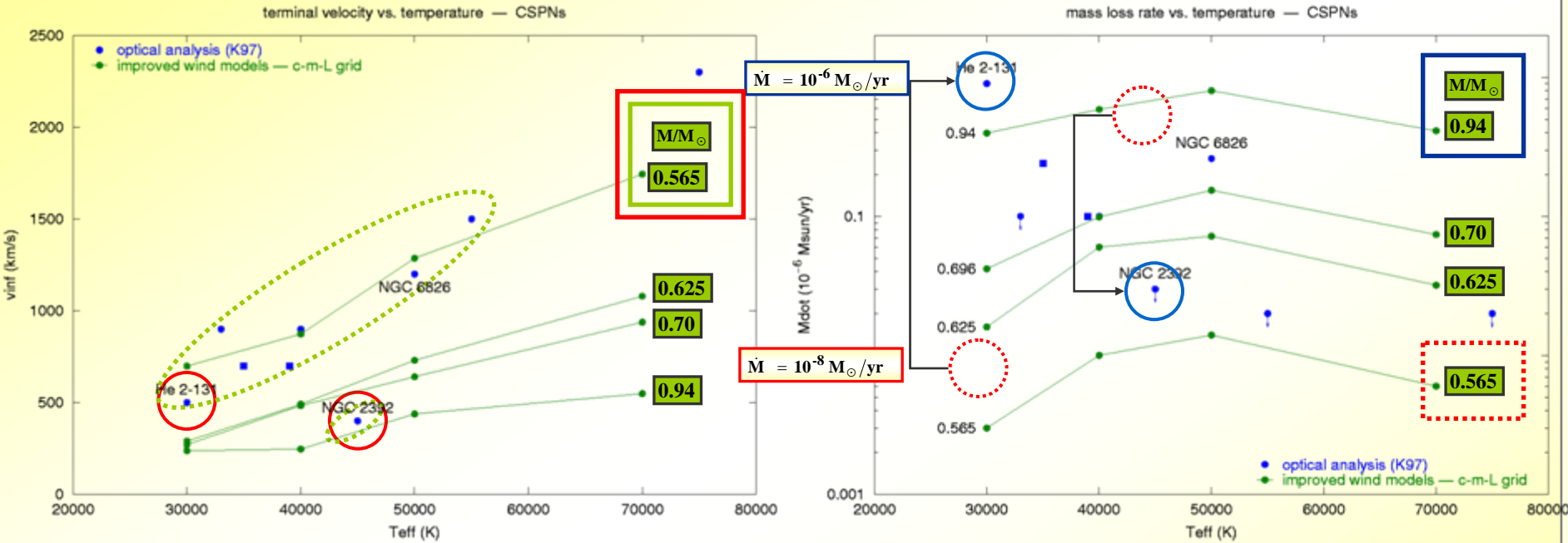


$$\dot{M} v_{\infty} R^{1/2} \propto L^{3/2}$$

Pauldrach, Hoffmann, Mendez, 2004, A&A 419, 1111

Wind properties of CSPN's

wind models calculated along evolutionary tracks based on the
core-mass-luminosity relation



terminal velocities

indicate mostly small masses $< 0.6 M_{\text{sun}}$
apart from NGC 2392
for which a high mass of $\sim 0.9 M_{\text{sun}}$
is indicated

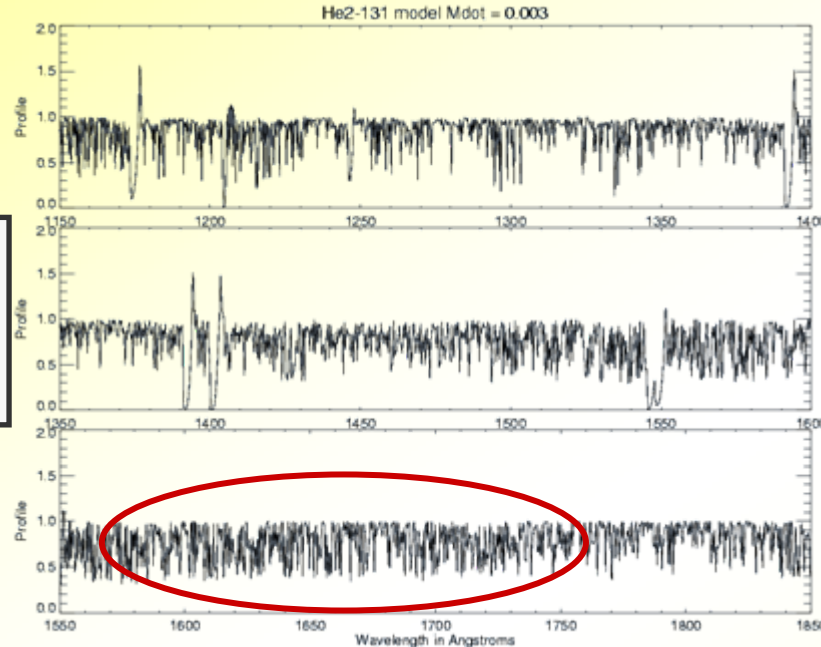
mass loss rates

indicate for 6 objects high masses $> 0.7 M_{\text{sun}}$
but for NGC 2392
a small mass of $< 0.6 M_{\text{sun}}$
is indicated

next step:

UV-diagnostics with models based on stellar parameters
consistent with the terminal velocities

Analysis of the UV-spectrum of He2 131



wind model based on stellar parameters consistent with the evolutionary tracks and the terminal velocity

$$T_{\text{eff}} = 30 \text{ kK}$$

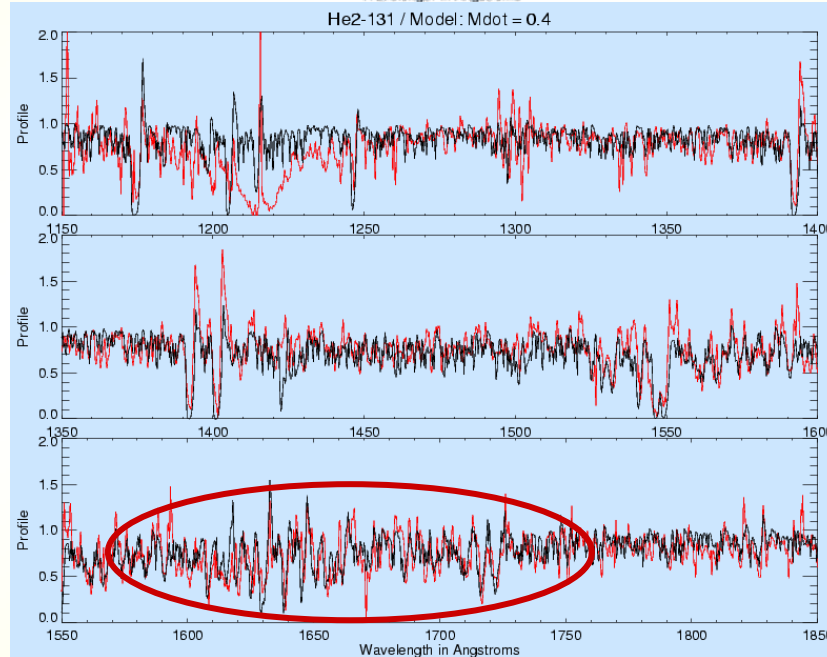
$$\log L/L_{\odot} = 3.59$$

$$\log g = 3.47$$

$$R/R_{\odot} = 2.3$$

$$\frac{\dot{M}}{10^{-6} M_{\odot}/\text{yr}} = 0.003$$

$$v_{\infty}/(\text{km/s}) = 470.$$



UV-diagnostics:
stellar parameters deduced from a comparison of observed and synthetic spectra

$$T_{\text{eff}} = 33 \text{ kK}$$

$$\log L/L_{\odot} = 4.51$$

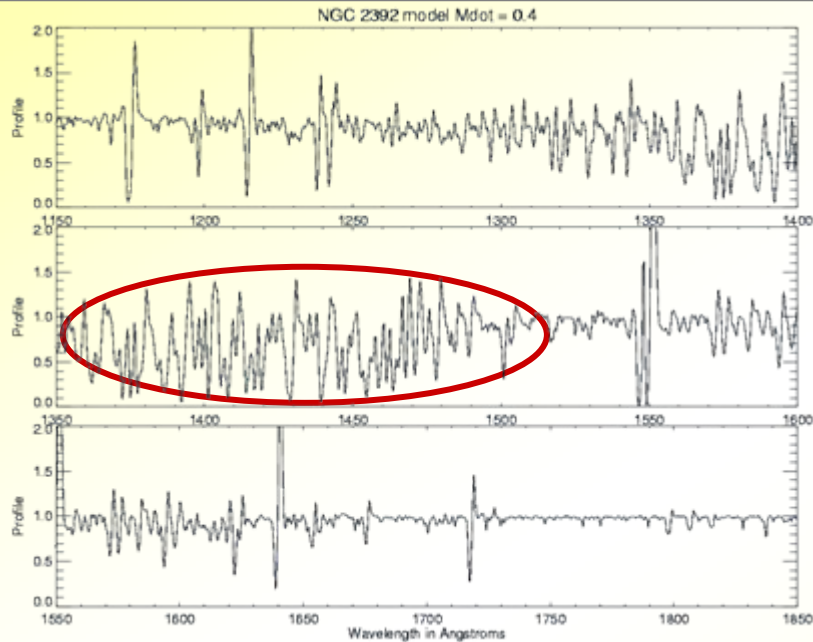
$$\log g = 3.1$$

$$R/R_{\odot} = 5.5$$

$$\frac{\dot{M}}{10^{-6} M_{\odot}/\text{yr}} = 0.35$$

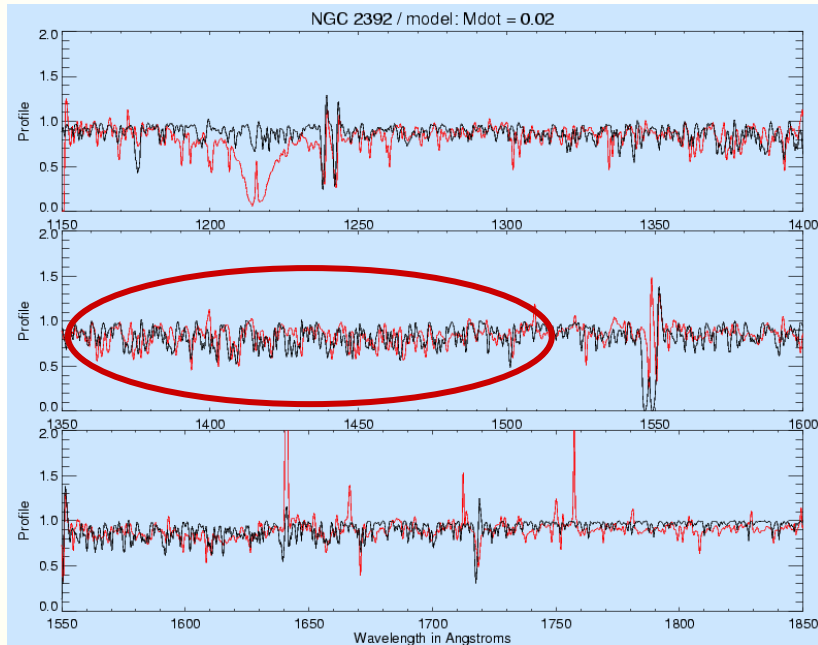
$$v_{\infty}/(\text{km/s}) = 450.$$

Analysis of the UV-spectrum of NGC 2392



wind model based on stellar parameters consistent with the evolutionary tracks and the terminal velocity

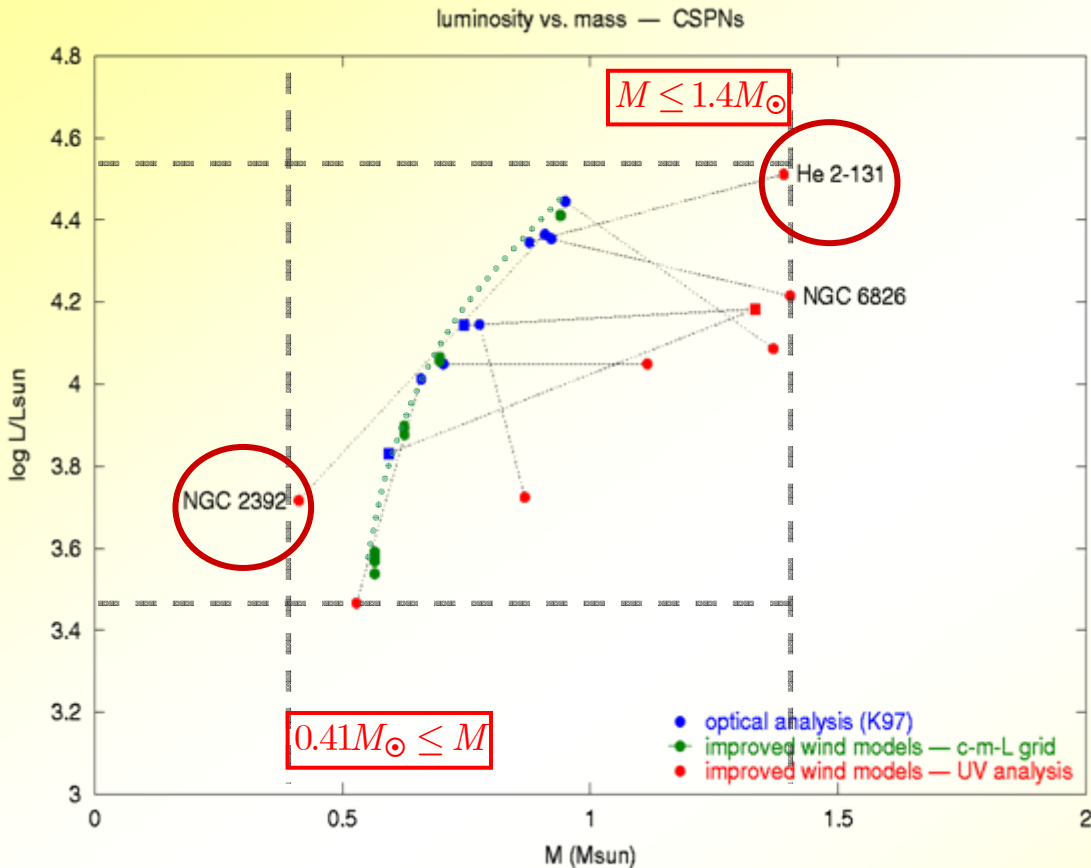
T_{eff}	=	45 kK
$\log L/L_{\odot}$	=	4.365
$\log g$	=	3.6
R/R_{\odot}	=	2.5
$\frac{\dot{M}}{10^{-6}M_{\odot}/\text{yr}}$	=	0.38
$v_{\infty}/(\text{km/s})$	=	400.



UV-diagnostics:
stellar parameters deduced from a comparison of observed and synthetic spectra

T_{eff}	=	40 kK
$\log L/L_{\odot}$	=	3.72
$\log g$	=	3.7
R/R_{\odot}	=	1.5
$\frac{\dot{M}}{10^{-6}M_{\odot}/\text{yr}}$	=	0.018
$v_{\infty}/(\text{km/s})$	=	420.

Luminosities and masses of the CSPN sample



luminosities are in the expected range

Object	T_{eff} (K)	R (R_{\odot})	$\log \frac{L}{L_{\odot}}$	M (M_{\odot})	\dot{M} ($10^{-6} M_{\odot}/\text{yr}$)	v_{∞} (km/s)
NGC 2392	40000	1.5	3.7	0.41	0.018	420
NGC 3242	75000	0.3	3.5	0.53	0.004	2400
IC 4637	55000	0.8	3.7	0.87	0.019	1500
IC 4593	40000	2.2	4.0	1.11	0.062	850
He 2-108	39000	2.7	4.2	1.33	0.072	800
Tc 1	35000	3.0	4.1	1.37	0.021	900
He 2-131	33000	5.5	4.5	1.39	0.35	450
NGC 6826	44000	2.2	4.2	1.40	0.18	1200

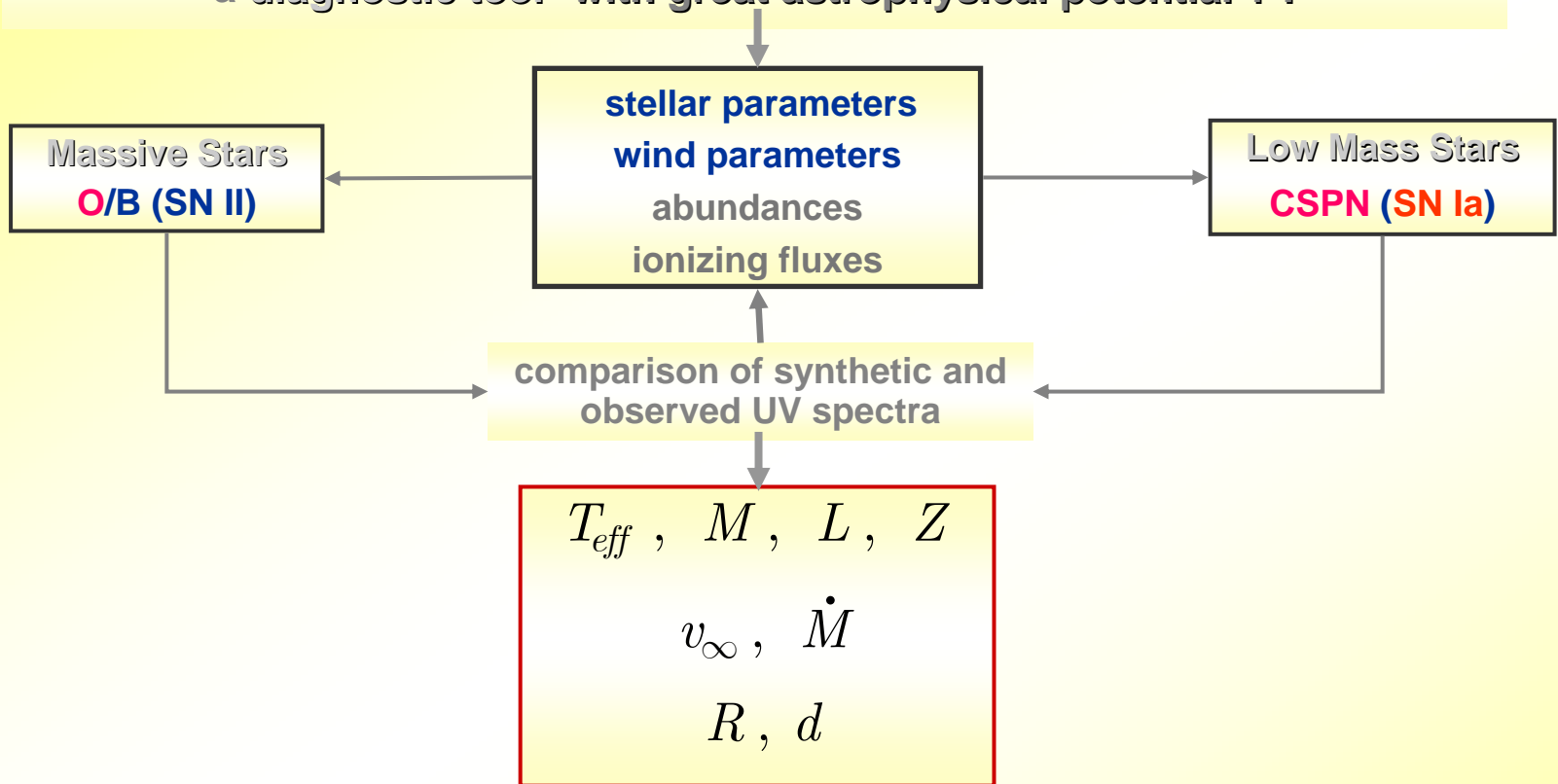
Object	T_{eff} (K)	R (R_{\odot})	$\log \frac{L}{L_{\odot}}$	M (M_{\odot})	\dot{M} ($10^{-6} M_{\odot}/\text{yr}$)	v_{∞} (km/s)
NGC 2392	45000	2.5	4.4	0.91	≤ 0.03	400
NGC 3242	75000	0.6	4.0	0.66	≤ 0.02	2300
IC 4637	55000	1.3	4.1	0.78	≤ 0.02	1500
IC 4593	40000	2.2	4.0	0.70	0.1	900
He 2-108	35000	3.2	4.1	0.75	0.24	700
Tc 1	33000	5.1	4.4	0.95	≤ 0.1	900
He 2-131	30000	5.5	4.3	0.88	0.9	500
NGC 6826	50000	2.0	4.3	0.92	0.26	1200

**very large spread in the masses
(0.4–1.4 Msun)**

**for the sample of CSPN
chosen there is no
well-defined relation
between
mass and luminosity !**

Conclusions

Consistent state of the art models for **expanding atmospheres of hot stars**
a diagnostic tool* with great astrophysical potential ? !



* download of the program package -including an easy to use interface- is possible from
<http://www.usm.uni-muenchen.de/people/adi/adi.html>